Çankaya University – ECE Department – ECE 376

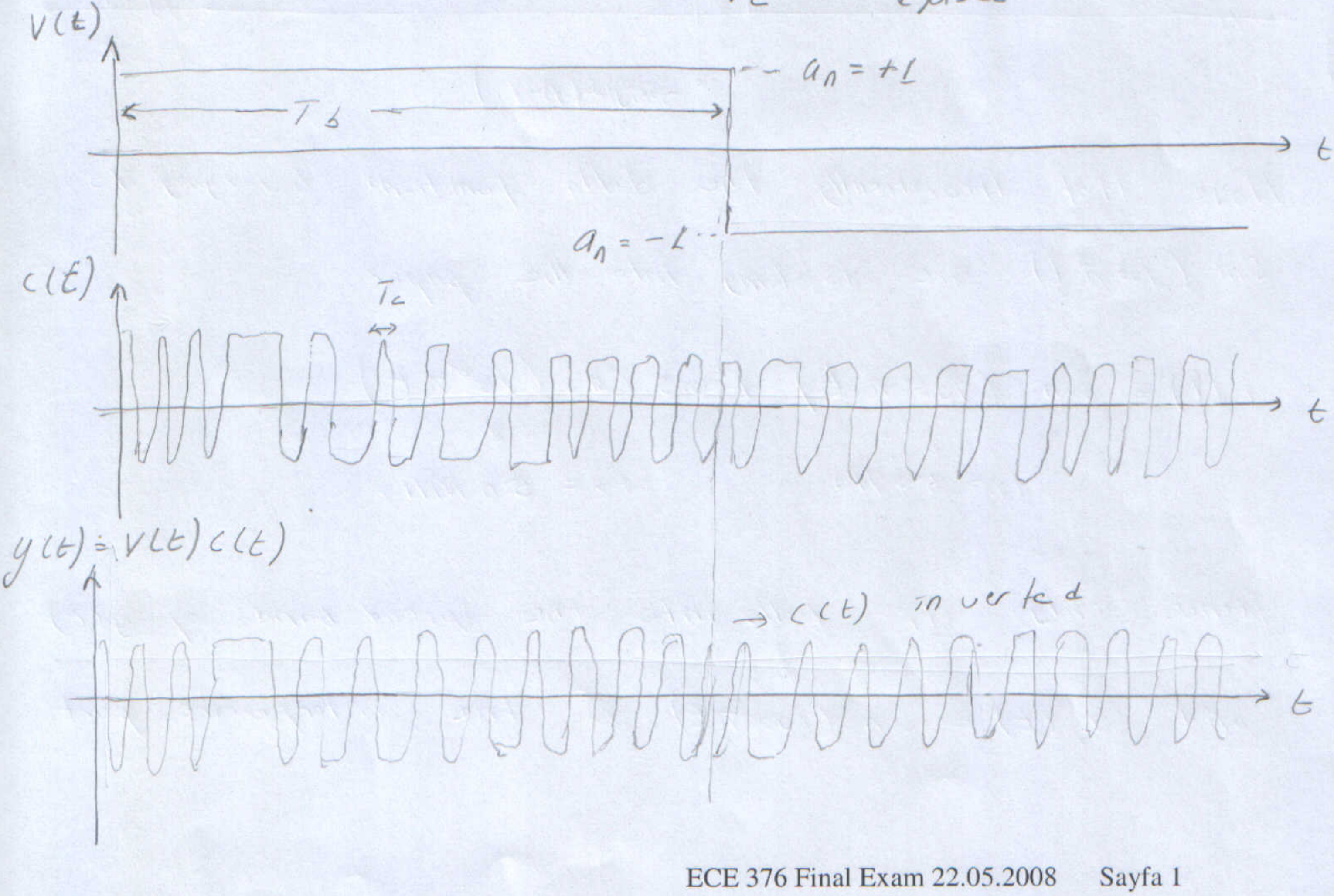
Student Name : Duration : 2 hours
Student Number : Open book exam

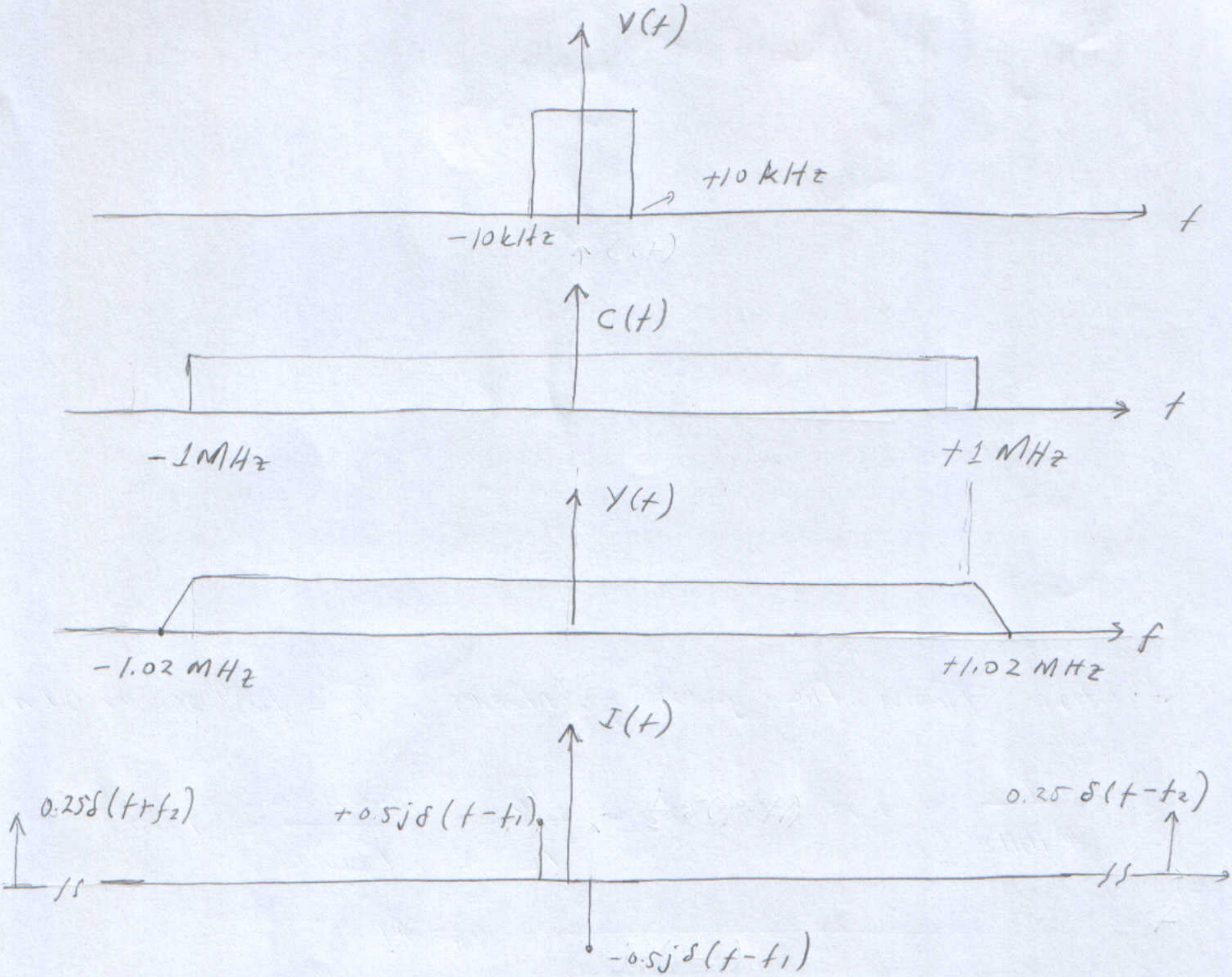
Questions

1. (35 Points) A binary antipodal PSK signal of $v(t)=\sum_{n=-\infty}^{\infty} a_n g_T\left(t-10^{-4}n\right)$, where $a_n = \mp 1$ and $g_T(t)=\begin{cases}1 & \text{for } 0<t<10^{-4} \text{ sec.}\\ 0 & \text{otherwise}\end{cases}$, is multiplied by a PN sequence that has 1 MHz bandwidth. Assuming that bandwidth is inversely proportional to symbol duration, determine T_b, T_c, L_c. Plot the approximate related time waveforms and the spectrums. To the spread signal, an interfering signal of $i(t)=\sin(100\pi t)+0.5\cos\left(10^8\pi t\right)$ is added. Find ***SNR*** as the ratio of signal energy to interference signal spectral density, i.e. $SNR = 2\mathcal{E}_b / I_0$. Comment on your results.

Solution: From the given quantities, $T_b = 10^{-4}$ sec or 0.1 msec.

$T_c = \frac{1}{1\,MHz} = 1\,\mu sec.$ $L_c = \frac{T_b}{T_c} = \frac{100\,\mu sec}{1\,\mu sec} = 100$





Note that theoretically the delta functions belonging to $\sin(100\pi t)$ are looking into the paper.

$$i(t) = \sin(100\pi t) + 0.5\cos(10^8\pi t)$$

$f_1 = 50\ Hz$ $\qquad$ $f_2 = 50\ MHz$

Hence only f_1 falls into the spread band of $y(t)$

and $i_1(t) = \sin(100\pi t)$ is the interference part.

Addendum to Q1 of ECE 376 Final 22.05.2008

Now $\mathcal{E}_b = \int_0^{T_b} [a_n\, g_T(t)]^2 dt$

when $g_T(t) = 1$

$= t \Big|_0^{10^{-4}} = 10^{-4}$ Joule

$$I_0 = \frac{\text{Power of } i_1(t)}{W\ (1.02\ MHz)} = \frac{1/2}{1.02 \times 10^{+6}}$$

$= 0.4902 \times 10^{-6}$ W/Hz (Joule)

$$SNR = \frac{2\,\mathcal{E}_b}{I_0} = \frac{2 \times 10^{-4}}{0.4902 \times 10^{-6}} = \frac{200}{0.4902} \simeq 408 \text{ or } 26 \text{ dB}$$

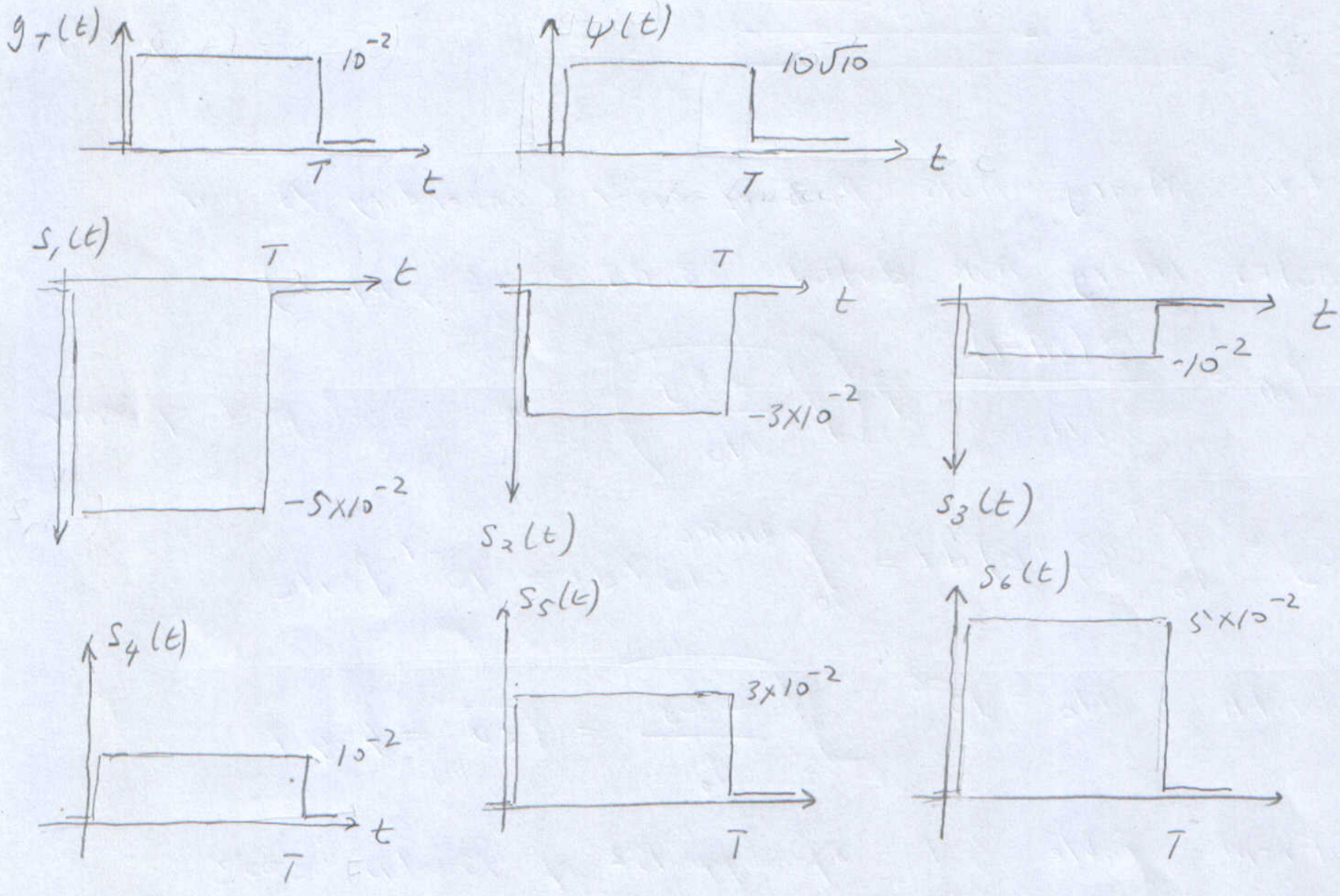
As seen the effect of interfering signal is minimal since SNR of 26 dB is a favourable figure.

2. (35 Points) The following signal set is given $s_4(t) = g_T(t)$, where

$$g_T(t) = \begin{cases} 10^{-2} & \text{for } 0 < t < 10^{-3} \text{ sec.} \\ 0 & \text{elsewhere} \end{cases}, \quad s_3(t) = -g_T(t),\ s_2(t) = -3g_T(t),\ s_1(t) = -5g_T(t),$$

$s_5(t) = 3g_T(t)$, $s_6(t) = 5g_T(t)$. What is the modulation type represented by this signal set ?. Identify M and N in this signal set. Draw the waveforms, the corresponding basis functions and the constellation diagram. For a AWGN with $N_0 = 10^{-8}$ W/Hz, what would be the (symbol) probability of error. ?

Solution: Since all signals are simple multiples of the basic waveform $g_T(t)$ (also related to $\varphi(t)$ as shown below) over the entire duration of $T = 10^{-3}$ sec or 1 msec., this is clearly ASK where $M = 6$, $N = 1$.



Therefore time waveforms may simply be written in terms of basis function with $g_T(t)$ replaced by $\varphi(t)$, as shown below

$s_1(t) = -0.5\times10^{-2.5}\varphi(t)$, $s_2(t) = -0.3\times10^{-2.5}\varphi(t)$, $s_3(t) = -0.1\times10^{-2.5}\varphi(t)$

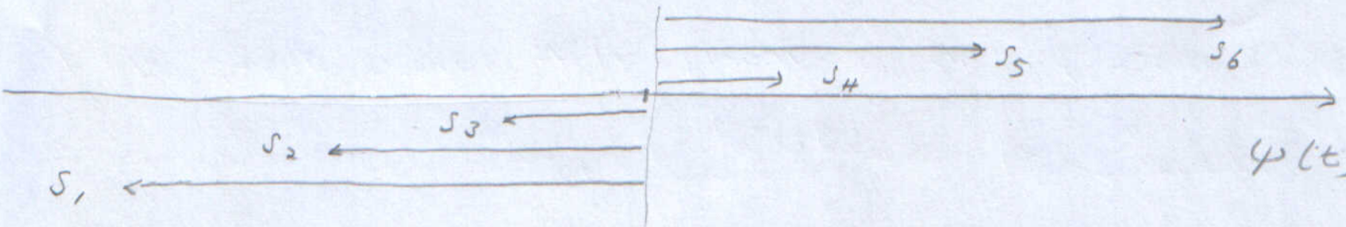
$s_4(t) = 0.1\times10^{-2.5}\varphi(t)$, $s_5(t) = 0.3\times10^{-2.5}\varphi(t)$, $s_6(t) = 0.5\times10^{-2.5}\varphi(t)$

Vectorwise

$s_1 = [-5\times10^{-3.5}]$, $s_2 = [-3\times10^{-3.5}]$, $s_3 = [-1\times10^{-3.5}]$, $s_4 = [1\times10^{-3.5}]$

$s_5 = [3\times10^{-3.5}]$, $s_6 = [5\times10^{-3.5}]$

Constellation Diagram



For Mary ASK Prob of error (according to our notes Mary ASK dated 3.05.2005, pp.2)

$$P_M = \frac{2(M-1)}{M} Q\left(\sqrt{\frac{2\mathcal{E}_g}{N_0}}\right) \quad \Leftarrow \text{Symbol prob. of error}$$

$$\mathcal{E}_g = \int_0^T g_T^2(t)\,dt = \int_0^{1msec} 10^{-4}\,dt = 10^{-7} \text{ Joule}$$

With $N_0 = 10^{-8}$ $\sqrt{\frac{2\mathcal{E}_g}{N_0}} = \sqrt{20} = 4.47$

From Table 4.1 of pp.152 of Proakis 2002

$$Q(4.47) = 4.5\times10^{-6}$$

$$P_M = \frac{2\times5}{6}\times4.5\times10^{-6} = 7.5\times10^{-6}$$

3. (30 Points) Answer the following questions as **True** or **False.** For the **False** ones give the correct answer or the reason. For the **True** ones, justify your answer.

a) PN sequences of different users in DS spread spectrum systems overlap along time axis :

True, separation of signals belonging to different users is established due to orthogonality of PN sequences (i.e multiplexing along time axis)

b) In DSB-SC, there is no noise performance improvement from the input to receiver to output :

True, in DSB-SC (input) SNR = SNR (output)

c) FM is preferred to PM because of its noise performance improvements :

True, in PM parabolic noise spectrum is not observed like the one in FM during demodulation

d) In FSK, message signals are always orthogonal to each other :

False, the orthogonality of message signals is measured with the parameter named γ_{mn} [Eq. (7.4.9) and Fig 7.26 on pp. 364 of Proakis 2002 →

e) MF detector is better than correlation type of detector :

False, MF detector is just an alternative representation of correlation type of detector. Both act as optimum detectors.

f) We sample the output of MF at $t = T$, since at that instance, the output is maximum :

False, we sample at $t = T$ since we start to integrate from $t = 0$, assuming that a symbol is wholly contained within the symbol duration (i.e the channel is $c(t) = 1$ or $c(t) = \delta(t)$) and prefer to make a decision →

e)
Accordingly orthogonality is acheived when Δf (separation between signal components) $= \frac{1}{2T}$

where T is symbol duration

f)
after having observed the whole behaviour of the received signal within its symbol duration.